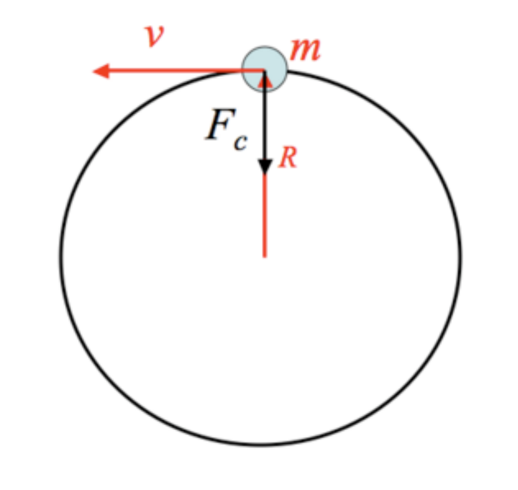
Circular Motion

Level 1 Physics



What you need to know

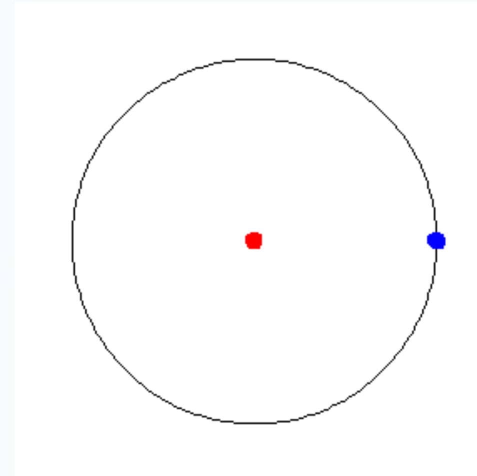
Objectives

- Explain the characteristics of uniform circular motion
- Derive the equation for centripetal acceleration of an object moving in a circle at constant speed
- Understand that centripetal force is not a new type of force
- Understand that centrifugal force does not exist
- Explain and apply the relationship between the speed and the orbital radius of a satellite
- Demonstrate proficiency in solving problems involving apparent weightlessness in a satellite and in an elevator

Essential Questions

- What are the applications of circular motion?
- What is the difference between centripetal and centrifugal force and is centrifugal force real ?
- What forces keep satellites in orbit?
- What evidence is there that a falling apple and an orbiting planet are identical situations?
- How does apparent weight vary during circular motion?

Velocity



Circumference – Distance an object covers in **ONE** revolution

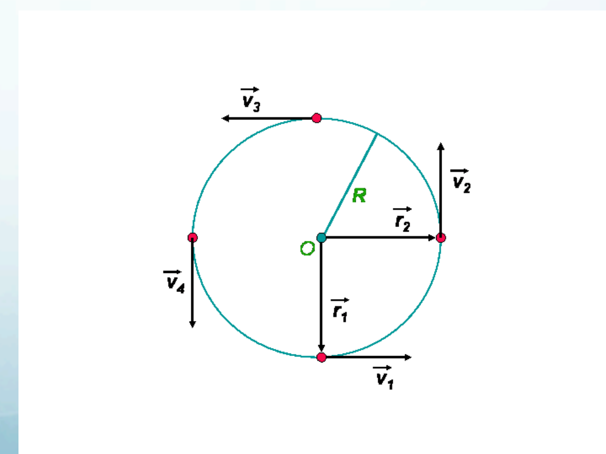
Period (T) – The time for **ONE** revolution

$$v = \frac{\Delta x}{\Delta t} = \frac{2\pi r}{\mathrm{T}}$$

In Uniform Circular Motion, speed is **constant!**

Velocity is **NOT** constant. The direction always changes at every point along the circle

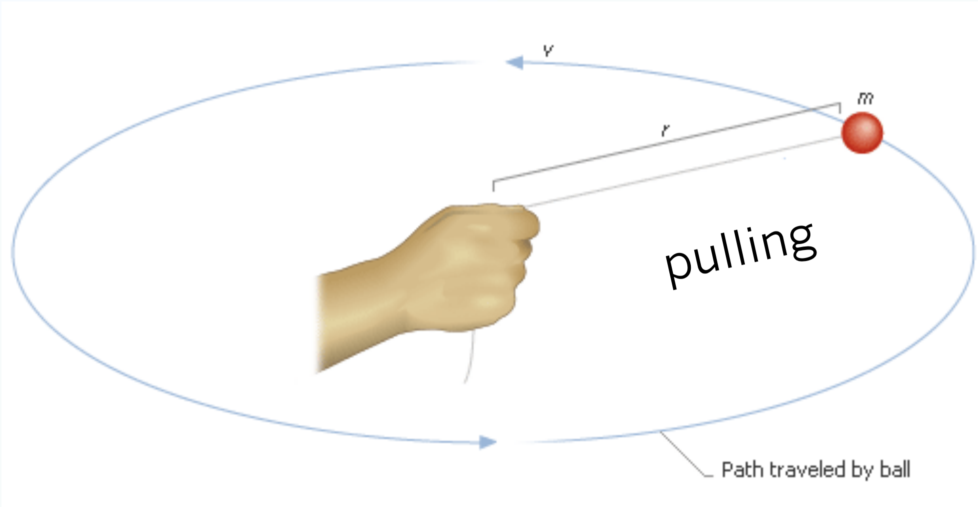
Velocity is **TANGENT** at every point along the circle



Motion in a Circle

Some important facts:

1. Velocity is a **VECTOR**
2. Vectors have magnitude **AND** Direction
3. Acceleration is defined as the **RATE of CHANGE of VELOCITY!**
4. According to Newton's second Law. The acceleration is **DIRECTLY** proportional to the force. $F_{net} \alpha$ acc



What can we conclude?

•If it is moving in a circle, the **DIRECTION** of the velocity is changing

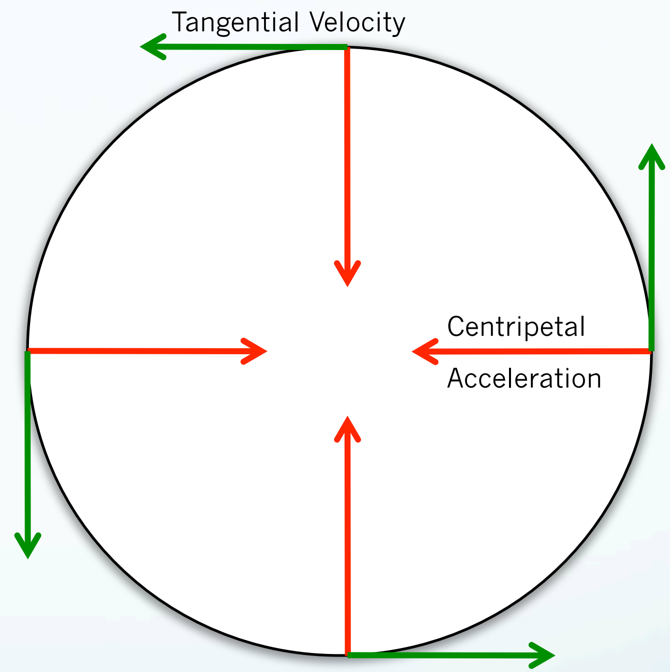
•If the velocity is changing, we have an acceleration

•Since we are **PULLING** towards the **CENTER** of the CIRCLE, we are applying a **NET FORCE** towards the CENTER.

•Since we have a NET FORCE we MUST have an **ACCELERATION.**

Centripetal Acceleration

This inward acceleration is defined as the centripetal acceleration. The word centripetal means "**Center Seeking**"



The magnitude of this acceleration can be described in 2 ways:

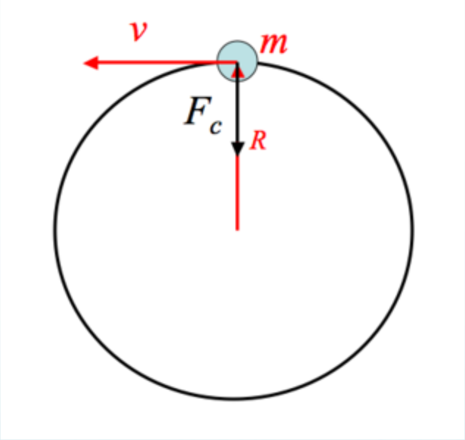
$$a_c = \frac{v^2}{R}$$

$$a_c = \frac{4\pi^2 R}{T^2}$$

U.C.M. and The Laws

Remember N.S.L., the acceleration is directly proportional to the force

Since the acceleration and the force are directly related, the force must ALSO point towards the center. This is called CENTRIPETAL FORCE.



$$\Sigma F = ma$$

$$a_c = \frac{v^2}{R}$$

$$\Sigma F = m\frac{v^2}{R}$$

$$F_c = \Sigma F = m\frac{v^2}{R}$$

NOTE: The centripetal force is a NET FORCE. It could be represented by one or more forces. So NEVER draw it in an F.B.D.

Example

A Ferris wheel with a diameter of 18.0 meters rotates 4 times in 1 minute. a) Calculate the velocity of the Ferris wheel. b) Calculate the centripetal acceleration of the Ferris wheel at a point along the outside. c) Calculate the centripetal force a 40 kg child experiences.

$$v_c = \frac{2\pi r}{T} = \frac{2(3.14)9}{15} =$$ **3.77 m/s**

$$a_c = \frac{v^2}{r} \rightarrow \frac{v^2}{9} =$$ **1.58 m/s/s**

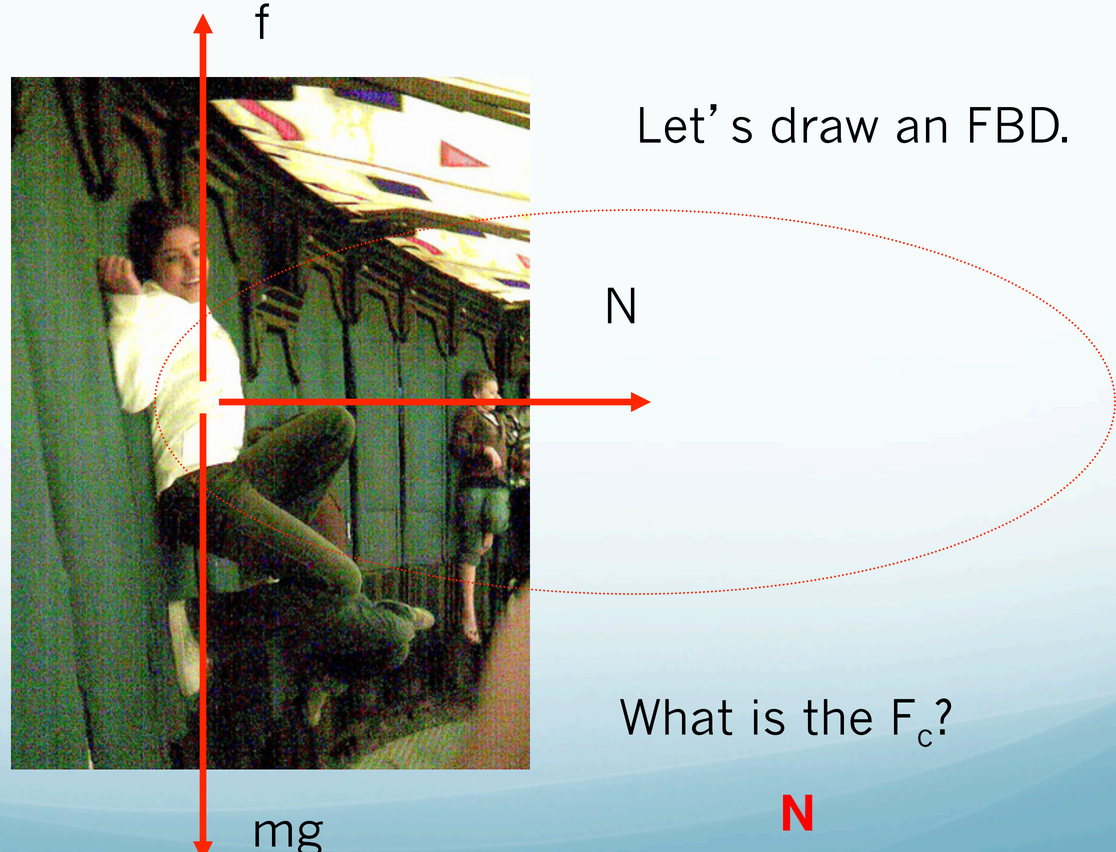
$$F_c = \frac{mv^2}{r} \rightarrow \frac{(40)v^2}{9} =$$ **63.17 N**

$$or \quad F_c = ma_c \rightarrow (40)(a_c) =$$ **63.17 N**

Centripetal Force and F.B.D's

The centripetal force is ANY force(s) which point toward the CENTER of the CIRCLE.

Turkish twist the ride



Let's draw an FBD.

What is the F_c?

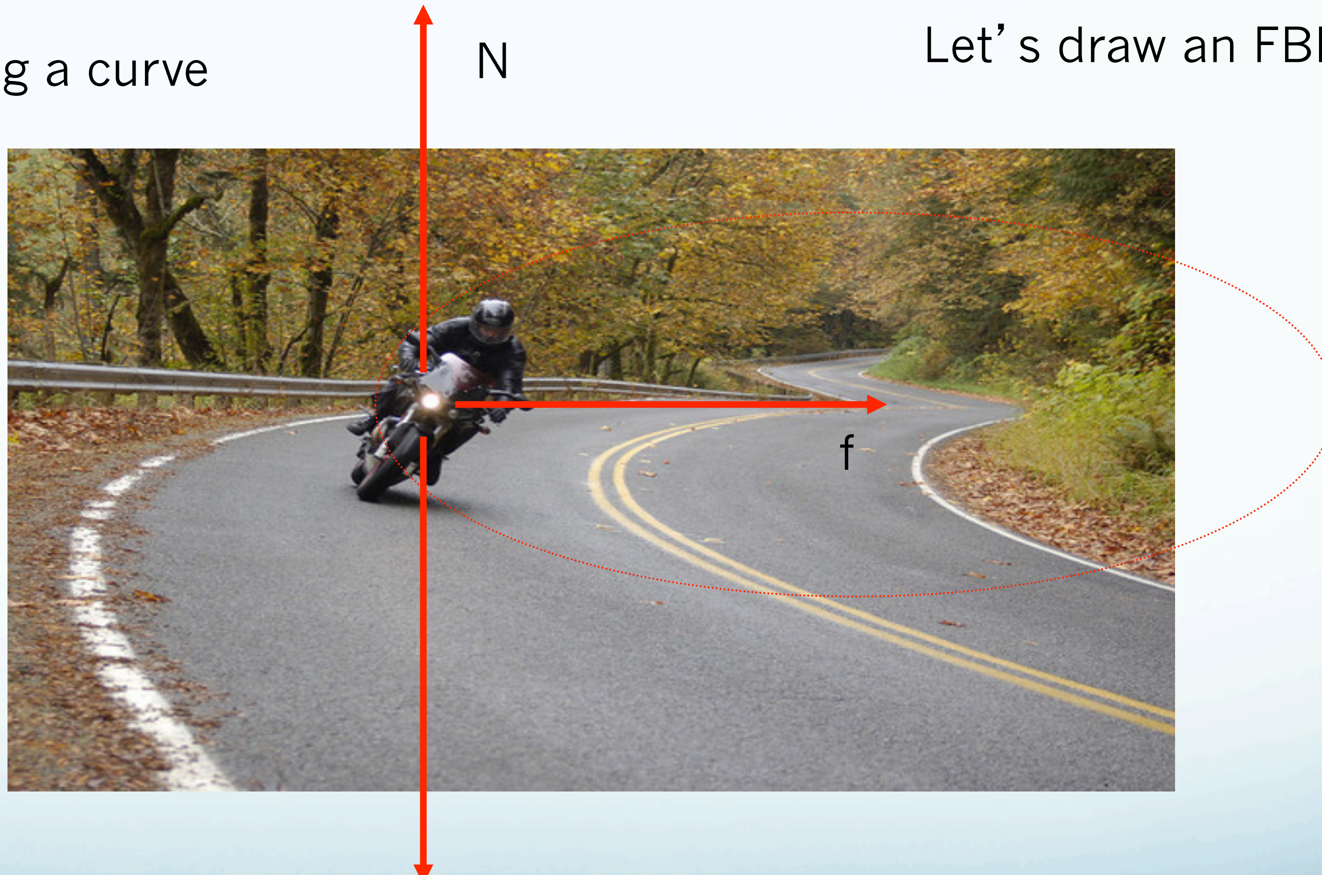
N

Time to Ride!

Centripetal Force and F.B.D's

Rounding a curve

Let's draw an FBD.

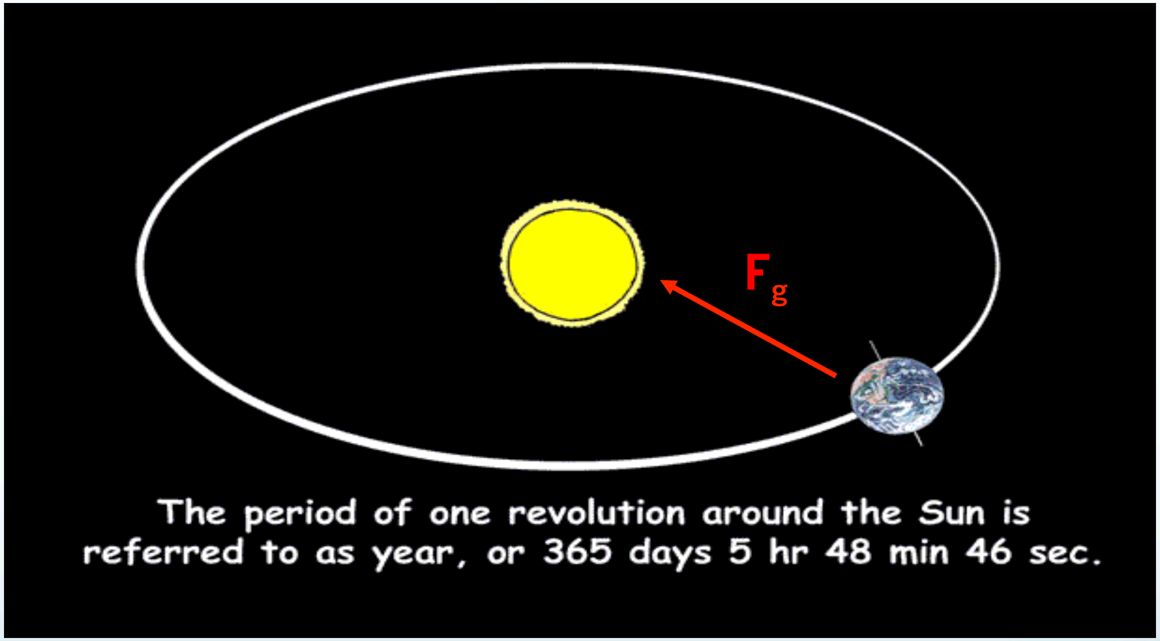


What is the F_c?

f

Centripetal Force and F.B.D’s

The earth in orbit around the sun

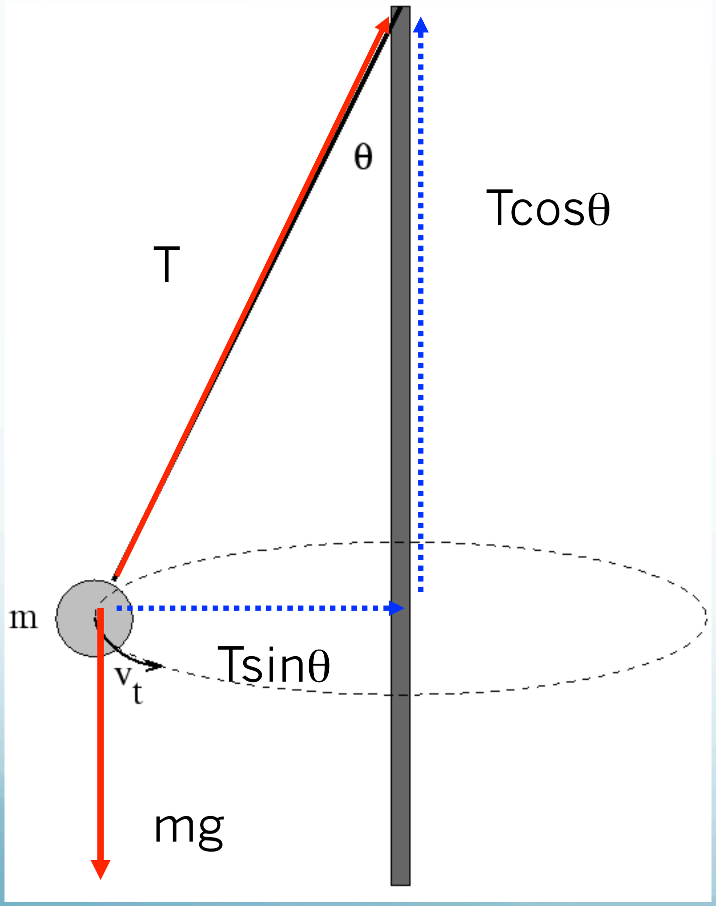


What is the Fc?

Fg

Centripetal Force and F.B.D's

Tether ball



What is the F_c?

Tsinθ

Satellites in Circular Orbit

- What is the force that keeps the satellites in orbit?
  - Gravitational pull of the Earth

$$\Sigma F = F_c = G\frac{mM_E}{r^2}$$

- To remain in orbit with a fixed radius
  - Only **ONE SPEED**!

$$F_c = G\frac{\cancel{m}M_E}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}}$$

Solve for v!

$$v^2 = \frac{GM_E}{r}$$

$$\boxed{v = \sqrt{\frac{GM_E}{r}}}$$

Problem

- Determine the speed of the Hubble Space Telescope orbiting at a height of 598 km above the earth's surface.

$$v = \sqrt{\frac{GM_E}{r}}$$

$$v = \sqrt{\frac{\left(6.67x10^{-11}\right)\left(5.98x10^{24}\right)}{6.98x10^{6}}}$$

$$v = 7.56x10^{3}\ \tfrac{m}{s}$$

Period of a Satellite

- Time required for ONE orbital revolution

$$v = \sqrt{\frac{GM_E}{r}}$$

Speed also equals

$$v = \frac{2\pi r}{T}$$

Set equations equal to one another and solve for the period

$$\frac{2\pi r}{T} = \sqrt{\frac{GM_E}{r}} \longrightarrow T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}$$

Problem

- What is the height H above the earth's surface at which all synchronous satellites (regardless of mass) must be placed in orbit?